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Nucleon form factors at low q

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Abstract. We discuss recent studies of the proton and neutron form factors at low momentum transfer. The *rms*-radii and Zemach moment, respectively, are determined from the *world* data set.

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Introduction

The root-mean-square (*rms*) radii of the proton and neutron are quantities of great interest for an understanding of the nucleon; they describe the most important integral properties concerning the size. Accurate knowledge of the *rms*-radii of the charge- and magnetization distributions are needed for the interpretation of high-precision measurements of transitions in hydrogen atoms, studied in connection with measurements of fundamental constants. These measurements recently have made great progress; the 2p-1s (HFS) splittings are known to 14 (12) significant digits. The interpretation of these quantities is now entirely limited by the accuracy with which the proton radii are known.

The proton *rms*-radii in the past in general have been determined from elastic electron-proton scattering. The usual approach has been to employ the most accurate cross sections at low momentum transfer q, and to perform an experimental separation of longitudinal (L, electric) and transverse (T, magnetic) form factors G_{ep} and G_{mp}. The resulting charge or magnetic data as a function of q^2 are then fit with an appropriate function to get the *rms*-radius , i.e. the $q^2 = 0$ slope of the form factor.

For the neutron, things are more complicated as first the contribution of the proton to electron-deuteron scattering has to be removed, hereby increasing the uncertainty. The subtraction of the proton contribution also amplifies greatly the effect of poorly controlled corrections such as FSI or MEC.

In the area of both the proton and the neutron radii, significant progress has been made recently, either by better analysis of existing data and/or new data. This progress is discussed in this paper.

Proton charge radius

The up to recently most accurate radius came for data mainly taken at Mainz [1]-[2]. After an L/T-separation via Rosenbluth plots, the data were fitted with a polynomial expansion of the form factor

$$G(q) = 1 - q^2\langle r^2\rangle/6 + q^4\langle r^4\rangle/120 - ...$$

The result: $r^e_{rms} = 0.862 \pm 0.012\ fm$. Fits based on dispersion relations and the Vector Dominance Model VDM [3,4] gave a lower radius, $0.847 \pm 0.009 fm$. The average, $0.854{\pm}0.012\ fm$ is quoted as the "best" value in CODATA [5]. When considering the systematic errors of the data, the error bar quoted is certainly significantly underestimated. Also, Coulomb distortion has not been included when fitting the cross sections.

A closer look at the "modelindependent" expansion of G_{ep} given above shows that this expansion has a severe drawback: once, at very low q, the $q^4\langle r^4\rangle$-term is small enough to not affect the result, also the $q^2\langle r^2\rangle/6$-term is small, and difficult to determine from the experimental form factors which are proportional to $1{-}q^2\langle r^2\rangle/6{+}...$ and subject to normalization errors. When including data at not-so-low q, which are also sensitive to the higher moments, one finds that, for exponential-type charge distributions (which apply to the proton), they give a large contribution to $G(q)$; the higher moments $\langle r^4\rangle = 2.5\ \langle r^2\rangle^2$, $\langle r^6\rangle = 11.6\ \langle r^2\rangle^3$ grow rapidly with order. As a consequence, there is no region of q where the lower moments can be determined *without* undue interference of the higher ones. Inclusion of the higher moments is limited by the convergence radius of the above expansion, which is only $\sim 1.4 fm^{-1}$.

Continued Fraction (CF) expansions

$$G(q) = \cfrac{1}{1 + \cfrac{q^2 b_1}{1 + \cfrac{q^2 b_2}{1 + \cdots}}}$$

are a special case of Padé approximants and have been introduced to find a function $f(z)$ specified by its moments $\langle z^n\rangle$ and to accelerate the convergence of poorly converging series [6]. The convergence radius of this expansion is much larger, and, for exponential-type densities, the b_1, b_2 coefficients (b_1 yielding the *rms*-radius) are nicely

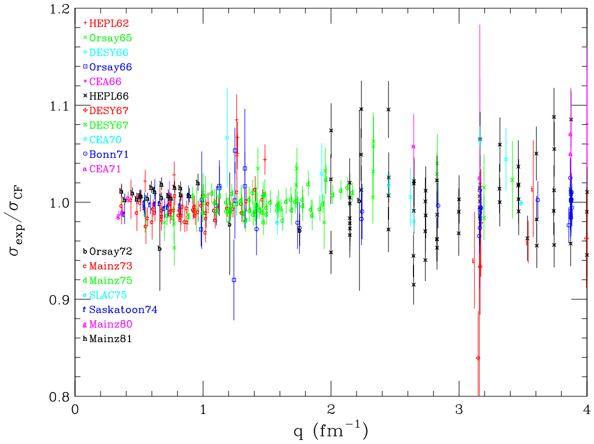

Fig. 1. The figure shows the ratio of experimental and fit e-p cross sections for the CF parameterization. Note the greatly expanded vertical scale

decoupled [7]. Tests of CF-expansion with pseudo data for $G_{ep}(q)$ and $G_{mp}(q)$, generated using known rms-radii, show that the arbitrariness in the choice of the number N of terms b_n (N=2 .. 5) used and the selection of a specific maximum momentum transfer for the data included, $q_{max} = 1 .. 5 fm^{-1}$, leads to a scatter of the fitted rms-radii of $\pm 0.010 fm$, this scatter covering both model and statistical error.

In order to determine the proton rms-radius we have used the world cross sections for $q < 4 fm^{-1}$. When parameterizing both $G_{ep}(q)$ and $G_{mp}(q)$ with the CF expansion and fitting G_{ep} and G_{mp} simultaneously to the cross sections, the L/T-separation is automatically performed, with a quality that is superior to the usual L/T-separation at individual values of q. The Coulomb corrections (usually omitted, but with significant effect) have been included according to [8,9].

The quality of the fits is quite good. We show in Fig. 1 the ratio of experimental cross sections and fit for the CF parameterization and 5 CF coefficients. When including the systematic uncertainty of the data – done by changing each data set by the quoted error, refitting and adding quadratically all the resulting changes – the result for the charge radius of the proton is $r^e_{rms} = 0.895 \pm 0.018 fm$ [10]. This radius is significantly larger than the values generally cited in the literature.

This larger value of the rms-radius now leads to agreement between calculated and experimental Lamb shift. The increase relative to earlier radii is understood as a consequence of now treating properly the higher moments $\langle r^n \rangle$.

A further improvement of the accuracy could be hoped to result from a μ-X-ray experiment performed at PSI [11]; at present, it is not clear, however, if the experiment was successful.

Proton magnetic radius and Zemach moment

In order to predict atomic hyperfine splittings HFS, an accurate knowledge on the magnetic structure is needed; the magnetic radius obviously also results from the above-described analysis of the e-p cross sections. More interesting, though, is the Zemach moment [12]: The bulk of the electron-nucleus magnetic interaction is short ranged and confined to the vicinity of the nucleus. This is also the only region of the electron's wave function that is significantly affected by the nuclear charge distribution, and the leading-order size effect on HFS was shown by Zemach[12] to depend on a convolution of charge- and magnetization densities

$$\Delta E_{\rm Zemach} = -2\, Z\, \alpha\, m\, \langle r \rangle_{(2)}\, E_{\rm F}$$

$$\begin{aligned}\langle r \rangle_{(2)} &= \int d^3 r\, r \int d^3 r' \rho_{\rm e}(|\mathbf{r} - \mathbf{r}'|) \rho_{\rm m}(r') \\ &= -\frac{4}{\pi} \int_0^\infty \frac{dq}{q^2} (G_{ep}(q^2) G_{mp}(q^2) - 1)\end{aligned}$$

where $E_{\rm F}$ is the Fermi hyperfine splitting, m is the electron mass, Z is the nuclear charge, α is the fine-structure constant.

One finds [13] that the Zemach moment depends on the detailed q-dependence of both $G_{ep}(q)$ and $G_{mp}(q)$; the usage of *e.g.* the dipole parameterization for both form factors, as has been assumed previously, does not lead to the correct answer. An advantage of the Zemach moment: it is sensitive to a higher q-range than needed for the determination of the rms-radii. As a consequence, the Zemach moment can be determined significantly more accurately than the rms-radius radius, $\langle r \rangle_{(2)} = 1.086 \pm 0.012 fm$, where the error bar includes statistics and all systematics. The smaller uncertainly results both from the anti-correlation of errors in G_{ep} and G_{mp} and the fact that at the larger q's the finite size effect is less sensitive to normalization errors of the data. Within 2σ the resulting HFS splitting calculated from the above Zemach moment agrees with experiment, the main uncertainty now coming from the difficult to calculate proton polarizability.

Neutron charge radius

In terms of the data base, little has changed on the neutron charge radius over the last years; the main effort has gone into measuring G_{en} at the larger values of q exploiting spin observables and (e,e'n) coincidence techniques. Two controversial points concerning the $q^2 \sim 0$ regime seem to have been clarified, though:

1. It for some time has been disputed whether the slope dG_{en}/dq^2 at $q^2 = 0$ reflects the genuine charge radius, or rather is mainly due to the 'Zitterbewegung' and the neutron anomalous magnetic moment, the Darwin-Foldy term. This question is very relevant for an understanding of the neutron, as the two interpretations lead to a negative or very small positive rms-radius squared, respectively. This question now is largely settled in favor of the former interpretation [14,15], as the Darwin-Foldy term is compensated by a relativistic correction to the $F_1(q)$ form factor. The neutron thus does have the structure expected from several models, a positive interior and a negative large-r tail.

2. The experimental slopes of the Dubna and Garching groups [16,17], measured via thermal neutron-heavy

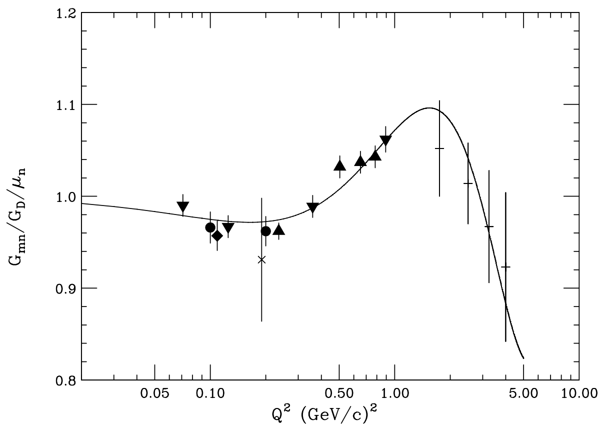

Fig. 2. The figure shows the ratio of experimental and dipole neutron magnetic form factor and the CF fit. The data are from d(e,e'n)/d(e,e'p) (▲▼), ^{3}**He**(e,e') (•,[18]) and d(e,e') (+,[19]). Note the greatly expanded vertical scale

atom scattering, differed by several σ; it is now understood that the use of the experimental neutron polarizability favors the Munich value, amounting to $\langle r^2 \rangle_n = -0.113 \pm 0.003 fm^2$.

Neutron magnetic radius

While the proton form factors are known with excellent precision, data for the neutron have been of much poorer quality due to the lack of a free neutron target. This is true for both the electric form factor and, to a somewhat lesser extent, for the magnetic one.

In the past, G_{mn} has been determined mostly from quasi-elastic $D(e,e')$ cross sections. The extraction of G_{mn} requires an L/T separation and a subtraction of the (dominant) proton magnetic contribution. The uncertainties resulting from the deuteron wave function, meson exchange currents (MEC), and final state interactions (FSI) are greatly amplified by the two subsequent subtractions and limited the accuracy of G_{mn} to ~20%. Several new methods have been used recently.

The best method to minimize the sensitivity to poorly controlled quantities is a determination of G_{mn} from the ratio $R = \sigma(e,e'n)/\sigma(e,e'p)$ on the deuteron in quasi-free kinematics [20,21,22]. The ratio is insensitive to the deuteron wave function and corrections due to FSI and MEC are calculable and small. The detection of the neutron ensures small contributions from e-p scattering, the remaining ones due to charge-exchange scattering can be removed easily via calculation. The price to pay is the need for a precise measurement of the *absolute* efficiency η of the neutron detector employed. A measurement of η and a detailed study of the detector response $\eta(x,y,E_n)$ as a function of the location of the impact of the neutron on the detector and neutron energy, however, is feasible when using the high-intensity tagged neutron beams one can produce at the proton-beam facilities [20,22].

The data that determine the neutron magnetic form factors with small (and well controlled) theoretical corrections are shown in Fig. 2. (We do not include the D(e,e'n)

Table 1. Electric and magnetic *rms*-radii of nucleons; [1] denotes that the Zemach moment is quoted, [2] refers to r^2_{rms}

particle	r^e_{rms}	r^m_{rms}
proton	0.895±0.018 fm	1.086±0.012 fm^1
neutron	–0.113±0.003 fm^2	0.873±0.015 fm

data obtained using efficiencies determined with tagging-reactions with a *tree*-body final state i.e. $D(e,p)e'n$ and $H(e,\pi)e'n$ [23,24], as, with only one charged particle detected, such reactions do not allow to properly tag the neutron [25]). A CF fit of that data [26] yields a magnetic *rms* radius for the neutron $r^m_{rms} = 0.873 \pm 0.015 fm$, with statistical and systematic errors included. This value reaches a precision that is comparable to (and even slightly better than) the one achieved for the proton charge radius.

A summary of the most accurate radii for the proton and neutron is given in Table 1.

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